DIFFUSION OF PARTICLES IN A HOMOGENEOUS PSEUDOFLUIDIZED BED

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The coefficients of longitudinal and transverse diffusion of the particles in a pseudofluidized bed are calculated for an arbitrary value of the Reynolds number characterizing the flow of the pseudofluidizing medium around the particles. The theory is compared with experiment.

The diffusion of fine suspended particles under homogeneous pseudofluidized conditions was considered in [1, 2] for a particle Reynolds number of $R<1$. However, in the majority of cases, the pseudofluidized beds encountered in actual practice (including the homogeneous variety) are characterized by values of R equal to several tens or even hundreds. In these cases the interaction of the pseudofluidizing medium with the particles is nonlinear, not only with respect to the concentration of the bed, but also with respect to the relative velocity of the suspended flow. In this paper we shall generalize the results of [1, 2] to pseudofluidized beds of comparatively coarse particles, for which R is high. The bed is assumed homogeneous in the sense that no "bubbles" filled solely with the pseudofluidizing medium or aggregates consisting of a large number of particles are formed in it. The particles in such a bed may be approximately considered as statistically independent.

We shall use a coordinate system in which the particles are, on average, at rest, and shall direct the x_1 axis along the average relative velocity of the suspended flow **u**. We shall regard this velocity and also the average volumetric concentration of the particles in the bed ρ as independent of coordinates and time. In this coordinate system the tensor representing the diffusion of the suspended particles due to their random psuedoturbulent pulsations is diagonal, and its eigenvalues may be expressed in the form [1]

$$D_i = \frac{\pi}{2}\int \Psi_{wi,wi}(0, \mathbf{k})\, d\mathbf{k} \tag{1}$$

where $\Psi_{wi,wi}(\omega, \mathbf{k})$ is the diagonal component of the tensor representing the spectral density of the random velocity of the particles $\mathbf{w}'$; ω and $\mathbf{k}$ are the frequency and wave vector of the pulsations. This quantity is in the usual way expressed in terms of the spectral gage dZ_W of the process $\mathbf{w}'$, which enters into its representation in the form of a stochastic Fourier–Stiltes integral. The equations for dZ_W, and also for the spectral gages dZ_V, dZ_p, dZ_ρ representing the pulsations of the velocity $\mathbf{v}'$ and the pressure p' of the liquid in the gaps between the particles and the pulsations of concentration ρ', are obtained from the stochastic equations relating to the pulsations in question. In the case under consideration these equations differ from those used in [1, 2] solely in that the expression for the spectral gage dZ_F of the pulsation in the force of interaction between the particles and the liquid $\mathbf{F}'$ is altered.

As in [1, 2], we use the results of Ergun [3] for the steady-state force acting on the particles in unit volume of the unperturbed bed; thus we have

$$F = d_0\rho(\beta_1 K_1 + \beta_2 K_2 u)\,\mathbf{u} \qquad (\rho \gtrsim 0.2-0.3)$$

$$\beta_1 = \frac{75}{2}\frac{\nu_0}{a^2}, \quad \beta_2 = \frac{1.75}{2}\frac{1}{a}, \quad K_1 = K = \frac{\rho}{1-\rho},$$

$$K_2 = 1, \quad \nu_0 = \frac{\mu_0}{d_0} \tag{2}$$

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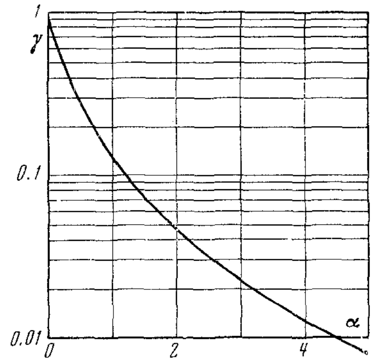

Fig. 1

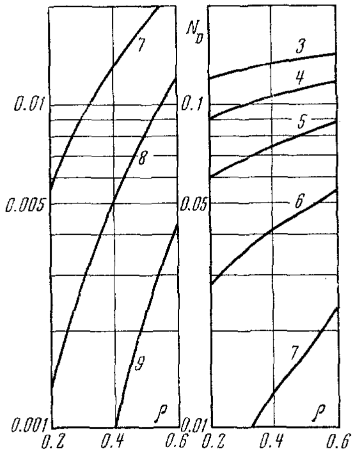

Fig. 2

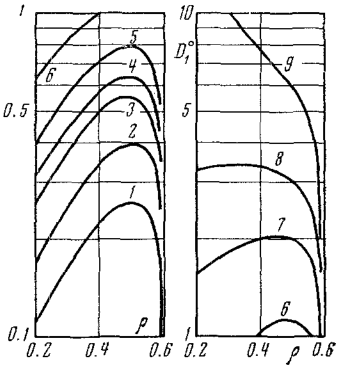

Fig. 3

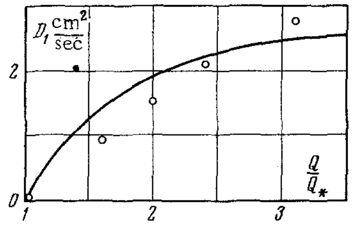

Fig. 4

where d_0, μ_0 are the density and viscosity of the liquid, a is the radius of the particles. The expression for $d\mathbf{Z}_F$ corresponding to (2) takes the form

$$d\mathbf{Z}_F = d_0\rho\,[(\beta_1 K + \beta_2 u)\,d\mathbf{Z}_u + \beta_2\,(\mathbf{u}_0 d\mathbf{Z}_u)\,\mathbf{u} + \beta_1 K'\mathbf{u}\,dZ_\rho] - i\mathbf{k}\rho\,dZ_p \tag{3}$$

$$d\mathbf{Z}_u = d\mathbf{Z}_v - d\mathbf{Z}_w, \quad \mathbf{u}_0 = \frac{\mathbf{u}}{u}, \quad K' = \frac{dK}{d\rho}$$

[The components of $d\mathbf{Z}_F$ due to different transient effects may here be ignored, since in order to determine D_i according to (1) it is sufficient to consider the equations for zero frequency $\omega = 0$ only.] If $\beta_2 = 0$, Eqs. (2) and (3) transform into those used in [2] for calculating the diffusion coefficients of small particles $(R < 1)$.

The equations for the spectral gages at $\omega = 0$ are written in the form [1, 2]

$$\mathbf{uk}\,dZ_\rho = (1 - \rho)\,\mathbf{k}\,d\mathbf{Z}_v, \qquad d\mathbf{Z}_F = 0$$

$$id_0\,(1 - \rho)\,(\mathbf{uk})\,d\mathbf{Z}_v = -i\mathbf{k}\,dZ_p - \mu_0 S\,[k^2 d\mathbf{Z}_v - {}^1/_3\mathbf{k}\,(\mathbf{k}\,d\mathbf{Z}_v)] \tag{4}$$

Here $S = S(\rho)$ is a function allowing for the deviation of the effective viscosity of the liquid filtering through the granular deposit from μ_0. (This function was also introduced in [1].)

From Eqs. (3) and (4) we derive an expression for $d\mathbf{Z}_W$

$$dZ_{wj} = \frac{dZ_\rho}{\beta_1 K + \beta_2 u\,(1 + \delta_{1j})}\left\{\beta_1 K' u_j + \frac{u}{1-\rho}\,\frac{k_1 k_j}{k^2}\,[\beta_1 K + \beta_2 u\,(1 + \delta_{1j}) + {}^4/_3 \nu_0 S k^2 + i\,(1-\rho)\,u k_1]\right\} \tag{5}$$

From this we obtain the representations for the diagonal components of the tensor representing the spectral density of the random vector $\mathbf{w}'$, valid for $\omega = 0$,

$$\Psi_{w1,\,w1}(0, \mathbf{k}) = \frac{1}{\varepsilon^2}\left(1 + \frac{3.5}{150}\frac{R}{\rho}\right)^{-2}\left\{\left[\frac{d\ln K}{d\rho} + \frac{k_1^2}{\varepsilon k^2} \times \left(1 + \frac{3.5}{150}\frac{R}{\rho} + \frac{8}{225}\frac{\varepsilon S}{\rho}\,ak)^2\right)\right]^2 + \left(\frac{1}{75}\frac{ak_1^3}{k^2}\frac{R}{\rho}\right)^2\right\}\Psi_{\rho,\rho}(0, \mathbf{k})$$

$$\Psi_{wj,wj}(0,\mathbf{k}) = \frac{1}{\varepsilon^2}\left(1+\frac{1.75}{150}\frac{R}{\rho}\right)^{-2}\left[\left(1+\frac{1.75}{150}\frac{R}{\rho}+\frac{8}{225}\frac{\varepsilon S}{\rho}(ak)^2\right)^2+\left(\frac{1}{75}\frac{ak_1\varepsilon R}{\rho}\right)^2\right]\frac{k_1^2k_j^2}{\varepsilon^2k^4}\Psi_{\rho,\rho}(0,\mathbf{k}) \quad (j=2,3) \quad (6)$$

$$\varepsilon = 1-\rho, \quad Q=\varepsilon u, \quad R = 2aQ\nu_0^{-1}$$

Here we have introduced the average porosity of the bed ε, the filtration velocity (volumetric rate of flow) of the liquid Q, and the Reynolds number R.

The function $S(\rho)$ for large R is unknown; however, as indicated in [1], this quantity has very little effect on the diffusion coefficients. Neglecting this function and also the relatively small terms in the brackets of (6), we obtain the approximate equation

$$\Psi_{w1,w1}(0,\mathbf{k}) \approx \frac{1}{\varepsilon^2}\left(1+\frac{3.5}{150}\frac{R}{\rho}\right)^{-2}\left[\frac{d\ln K}{d\rho}+\frac{k_1^2}{\varepsilon k^2}\left(1+\frac{3.5}{150}\frac{R}{\rho}\right)\right]^2 \times$$
$$\times Q^2\Psi_{\rho,\rho}(0,\mathbf{k}), \quad \Psi_{wj,wj}(0,\mathbf{k}) \approx \frac{1}{\varepsilon^4}\frac{k_1^2k_j^2}{k^4}Q^2\Psi_{\rho,\rho}(0,\mathbf{k}) \quad (j=2,3) \qquad (7)$$

For the spectral density of the quantity ρ' we use the expression derived in [4]. Then

$$\Psi_{\rho,\rho}(0,\mathbf{k}) = \frac{\Phi_{\rho,\rho}(\mathbf{k})}{\pi \mathbf{Dkk}}, \quad \Phi_{\rho,\rho}(\mathbf{k}) = \frac{\Phi}{k_0^3}Y(k_0-k)$$
$$\Phi = \frac{3}{4\pi}\rho^2\left(1-\frac{\rho}{\rho_*}\right), \qquad k_0^3 = \frac{9\pi\rho}{2}\frac{1}{a^3} \qquad (8)$$

where Y(x) is a Heaviside function and ρ_* is the concentration of the bed of particles in the close-packed state.

Proceeding with the calculation, we obtain the following equations from (1), (7), and (8):

$$D_1D_2 = \frac{2\pi\Phi}{k_0^2}\frac{\gamma^2Q^2}{\varepsilon^4}(\alpha^2J_0+2\alpha J_2+J_4) \qquad (9)$$
$$D_2^2 = \frac{\pi\Phi}{k_0^2}\frac{\gamma^2Q^2}{\varepsilon^4}(J_2-J_4), \quad J_n = \int_0^1\frac{t^n dt}{t^2+\gamma^2}, \quad D_3\equiv D_2$$

Here we have introduced the dimensionless parameters

$$\alpha = \varepsilon\frac{d\ln K}{d\rho}\left(1+\frac{3.5}{150}\frac{R}{\rho}\right)^{-1} = \frac{1}{\rho+0.233R}, \quad \gamma = \left(\frac{D_2}{D_1-D_2}\right)^{1/2} \qquad (10)$$

From Eqs. (9) we obtain a transcendental equation for γ and expressions for D_1, D_2, and D_3 in terms of the single positive root of this equation:

$$2\gamma^2(\alpha^2J_0+2\alpha J_2+J_4) = (1+\gamma^2)(J_2-J_4)$$
$$D_j = D_j^0 aQ \quad (j=1,2,3), \quad D_2^0\equiv D_3^0 = N_D D_1^0 \qquad (11)$$
$$D_1^0 = 0.358\frac{\rho^{2/3}}{(1-\rho)^2}\left(1-\frac{\rho}{\rho_*}\right)^{1/2}\frac{1+\gamma^2}{\gamma}(J_2-J_4)^{1/2}$$

For R = 0 these equations transform into those considered in [1, 2].

We note that Eqs. (11) describe the diffusion of particles not only in a psuedofluidized bed but also quite generally in flows of suspensions containing both fine and coarse particles, provided that the space and time scales of the average flow are much greater than the corresponding scales of the pseudoturbulent pulsations.

The solution of the first equation in (11) as a function of the parameter α from (10) is presented in Fig. 1. Figures 2 and 3 give the dependences of N_D and D_1^0 on ρ for $\rho_* = 0.60$ and different R values. (Curves 1-9 in Figs. 2 and 3 are plotted for values of R equal to ∞, 200, 100, 80, 60, 40, 20, 10 and 0 respectively.)

We see that the pseudoturbulent diffusion of the particles for small R is sharply anisotropic (N_D small), as already mentioned in [1]. However, N_D increases rapidly with increasing R; thus for R = 200 and R → ∞ it is practically independent of ρ and equals 0.226 and 0.420, respectively. Thus in a homogeneous pseudofluidized bed of fairly coarse particles the longitudinal diffusion is only 2.5–5 times more intensive than the transverse. With increasing R in the range (0–20) the maximum of the $D_1^0 = D_1^0(\rho)$ relationship moves rapidly in the large ρ direction; if $R \gtrsim 50$ the maximum is reached for $\rho \approx 0.5$. (We remember that in the calculations we used $\rho_* = 0.60$; for other ρ_* values the position of the maximum may of course change.)

Suppose that, at the instant at which the bed passes into the pseudofluidized state, the bed is characterized by a value of $R = R_*$. As the rate of flow Q increases, the concentration ρ diminishes monotonically, while the parameter R increases linearly. Hence points on different curves in Figs. 2 and 3 will represent different states of a bed composed of the same particles, pseudofluidized by the same liquid. It is accordingly of interest to plot the $N_D(\rho)$, $D_1^0(\rho)$ relationships not only for fixed R, but also for different specific beds.

The literature contains a large number of indirect conclusions regarding the diffusion of particles [1], and the theory which has so far been developed is in general accord with these conclusions. However, there have been very few systematic and reasonably exhaustive investigations into pseudoturbulent diffusion which might provide a quantitative proof of the theory. We shall shortly consider the experiments of Carlos and Richardson [5], who determined, in particular, the coefficients of longitudinal diffusion of particles in a homogeneous bed of glass spheres ~0.9 cm in diameter, pseudofluidized by dimethyl phthalate (viscosity 0.1 P).

Carlos and Richardson [5] studied the dynamics of the spreading of a thin horizontal layer of tracer spheres introduced into a stationary packing, which was then pseudofluidized. The coefficient D_1 was determined by comparing the measured concentration profiles of the tracer spheres at different moments of time with the solutions of the Fick equation corresponding to different D_1. Two serious objections may be advanced against this method. Firstly, for fairly short times the diffusion is described, not by the Fick equation, but by a more complex equation of the hyperbolic type [4]. Secondly (and this is particularly important), under the experimental conditions of [5] there was an intensive circulation of the suspended material in the bed, so that the spreading of the tracer particles was due not so much to diffusion itself as to the convective transfer of the particles. (For comparison, we may indicate that the mean velocity of the rising motion of the particles in the central part of the bed W_1 equaled 1-4 cm/sec, while the filtration velocity Q_* at the onset of pseudofluidization was only 4.8 cm/sec.)

The existence of a rising flow of particles in the center of the bed and a descending flow at the walls under conditions of pseudofluidization similar to those employed in [5] may easily be seen from the photographs of the spreading of tracer particles in a bed presented in [6]. The important role of the convective transfer of the particles by the secondary circulatory flows is also emphasized in [7]. It follows that the values of D_1 obtained in [5] by the method indicated are much too high and should not be used to characterize true particle diffusion.

In addition to this, Carlos and Richardson [5] determined the component $\langle(\Delta x_1)^2\rangle$ of the mean square vertical displacement of the particles in a time Δt due to the pulsations of the particles. As we should expect, this quantity was proportional to Δt, thus enabling the true diffusion coefficient D_1 to be determined from the Einstein formula

$$\langle(\Delta x_1)^2\rangle = 2D_1\Delta t \tag{12}$$

If the trajectory of a uniquely distinguished particle is used in the averaging, this formula is only valid when Δt greatly exceeds the time scale of the pulsations. If the averaging is carried out over many particles (i.e., effectively over the whole aggregate), as in [5], Eq. (12) is valid for any Δt. The values of D_1 determined from (12) in the experiments of [5] are shown by the light circles in Fig. 4.

The characteristic scale ΔT of the time of measurement in [5] was 10 sec. The convective displacement of a particle in this time $X_c \approx W_1\Delta T \sim 10$-40 cm, while the corresponding mean square displacement due to diffusion $X_d \approx \sqrt{2D_1\Delta T} \sim 1$-5 cm, considerably smaller than X_c. This estimate clearly illustrates the inadequacy of the first method of determining D_1 in [5].

The close-packed state of the bed in the experiments of [5] corresponded to a simple cubic packing, so that $\rho_* = 0.524$. (This value may easily be obtained by converting the values of Q and ρ in [5] to the initial pseudofluidized state, when $Q = Q_*$.) The Reynolds number varied from 50 to 160 for a Q/Q_* variation from 1 to ~3.1. The theoretical relationship for D_1 calculated from (11) for the values of ρ_* and R indicated is also shown in Fig. 4. Considering the complication of the theory itself and the many difficulties which arise in setting up fine experiments with a pseudofluidized bed, and especially in the interpretation of these, the agreement between the theoretical and experimental data may be regarded as entirely satisfactory. We note that the general character of the curve in Fig. 4 is also supported by the data presented in [6]. Certain values of D_1 were also obtained in [7], using a model "two-dimensional" bed of hollow spheres 3.5 cm in diameter, pseudofluidized by air, which corresponded to extremely high values of R. Unfortunately not all the data required for a rigorous estimate of the results were presented in [7], particularly those needed for comparison with theory. However, it is clear that the curve representing the dependence of D_1 on Q/Q_* should in this case pass considerably higher than the curve in Fig. 4. One of the experimental points of [7] is shown as a black circle in Fig. 4 by way of illustration.

Further comparison of theory with experiment and any possible refinement of the theory are at present impeded by the lack of other reasonably reliable experimental data regarding the diffusion of particles in dispersed systems.

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